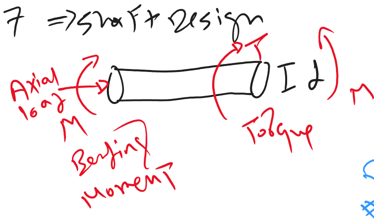
Tutorial 1

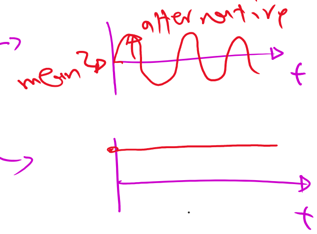
Friday, February 21, 2025 8:03 PM

Ch7 => Shaft Design



Dynamic

=> M
T
Bending stress
Torsion stress
"Stress"



$M_m = 0$, $M_a = ✓$ (Moment)

$T_m = ✓$, $T_a = 0$ (Torque)

Stress

$$\sigma_a = K_f \frac{32M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32M_m}{\pi d^3} \tag{7–2}$$

σ → Bending stress

$$\tau_a = K_{fs} \frac{16T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16T_m}{\pi d^3} \tag{7–3}$$

τ → Torsion stress

m → mean
a → amplitude

Distortion Energy

Von Mises

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2} \tag{7–4}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2} \tag{7–5}$$

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs}T_a)^2} \qquad B = \sqrt{4(K_f M_m)^2 + 3(K_{fs}T_m)^2} \tag{7–6}$$

($3(K_{fs}T_a)^2$ → 0; $4(K_f M_m)^2$ → 0)

=> Kf → Bending
=> Kfs → Torsion
Fatigue stress concentration

M_a => moment amplitude

T_m → mean torque

"Fatigue"

DE-Goodman

$$n = \frac{\pi d^3}{16}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} \tag{7–7}$$

$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]^{1/3} \tag{7–8}$$

DE-Morrow

$$n = \frac{\pi d^3}{16}\left(\frac{A}{S_e} + \frac{B}{\tilde{\sigma}_f}\right)^{-1} \tag{7–9}$$

$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{\tilde{\sigma}_f}\right)\right]^{1/3} \tag{7–10}$$

DE-Gerber

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\} \tag{7–11}$$

$$d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3} \tag{7–12}$$

DE-SWT

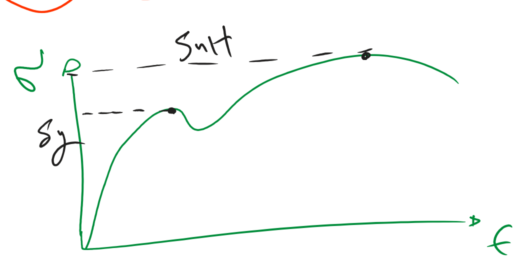
$$n = \frac{\pi d^3}{16}\frac{S_e}{(A^2 + AB)^{1/2}} \tag{7–13}$$

$$d = \left[\frac{16n}{\pi S_e}(A^2 + AB)^{1/2}\right]^{1/3} \tag{7–14}$$

A, B

d → diameter of shaft
n → fatigue factor of safety

Se => endurance limit
Sut → ultimate strength



n_y => yield factor of safety

S_y => yield strength

$$n_y = \frac{S_y}{\sigma'_{max}} \qquad (7\text{–}16)$$

$$\sigma'_{max} = [(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2]^{1/2}$$
$$= \left[\left(\frac{32K_f(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2} \qquad (7\text{–}15)$$

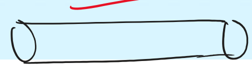
M_a ✓
T_m ✓

EXAMPLE 7–1

At a machined shaft shoulder the small diameter d is 27.94 mm, the large diameter D is 41.91 mm, and the fillet radius is 2.79 mm. The bending moment is 142.35 N · mm and the steady torsion moment is 124.28 N · mm. The heat-treated steel shaft has an ultimate strength of S_{ut} = 724 MPa, a yield strength of S_y = 565 MPa, and a true fracture strength of $\tilde{\sigma}_f$ = 1069 MPa. The reliability goal for the endurance limit is 0.99.

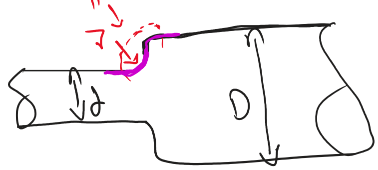
(*a*) Determine the fatigue factor of safety of the design using each of the fatigue failure criteria described in this section. "n" "4"

(*b*) Determine the yielding factor of safety. "n_y"



#step shaft

#shoulder



$M_m = 0$
$M_a = 142.35$
$T_m = 124.28$
$T_a = 0$

$d = 27.94$ mm
$D = 41.91$ mm
$r = 2.79$ mm

Bending moment $M = 142.35$ N.mm
torsion $T = 124.28$ N.mm
(steel) $S_{ut} = 724$ MPa
$S_y = 565$ MPa
$\tilde{\sigma}_f = 1069$ MPa
Reliability $R = .99$

Fatigue factor of safety (n)

$$n = \frac{\pi d^3}{16}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1}$$

$M_m = 0$
$M_a = 142.35$
$T_m = 124.28$

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs}T_a)^2} \qquad (7\text{–}6)$$

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$
$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2} \quad (7\text{–}6)$$

$M_a = 142.33$
$T_m = 124.28$
$T_a = 0$

"K_f" "K_{fs}" → fatigue stress concentration
⇒ "K_t" "K_{ts}" → stress concentration
Table (A-15)

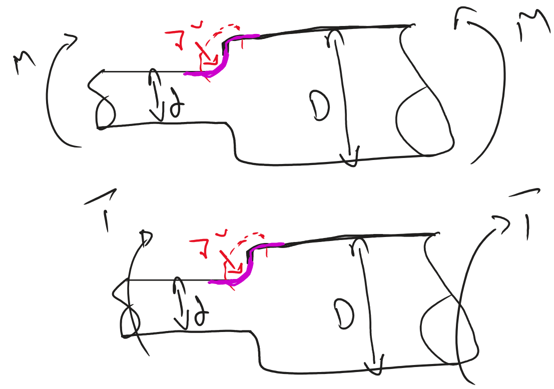
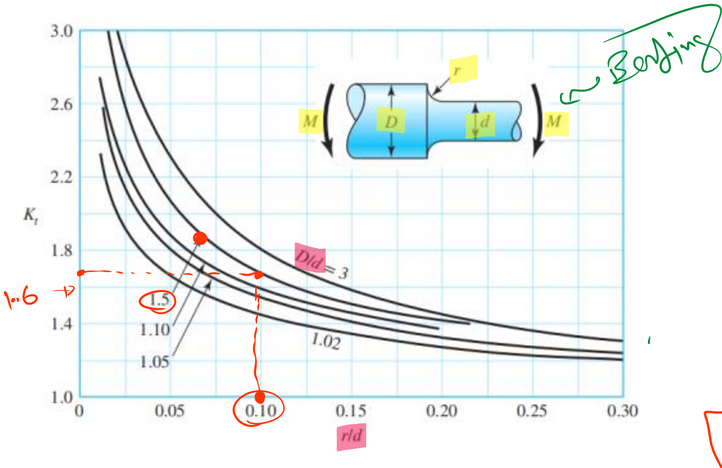


Figure A–15–9

Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

"K_t"



$d = 27.94$ mm
$D = 41.91$ mm
$r = 2.79$ mm

$\frac{D}{d} = \frac{41.91}{27.94} = 1.5$

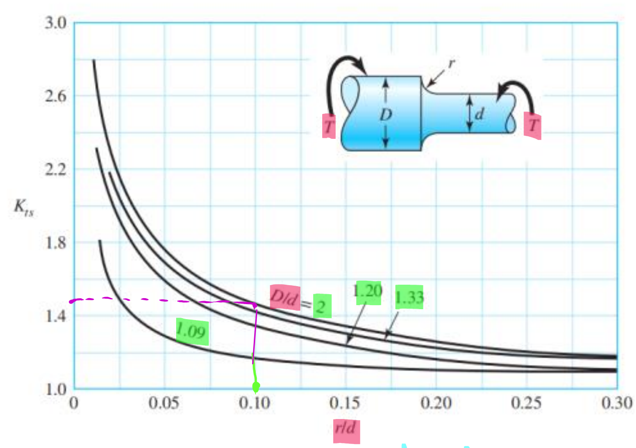
$\frac{r}{d} = \frac{2.79}{27.94} = .1$

$K_t = 1.68$

Figure A–15–8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

"K_{fs}"



$D/d = 1.5$
$r/d = .1$

"$K_{ts} = 1.42$"

3 Determine fatigue stress-concentration factor, K_f or K_{fs}. First, find K_t or K_{ts} from Table A–15.

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_s(K_{ts} - 1) \quad (6\text{–}32)$$

Obtain q from either Figure 6–26 or 6–27.

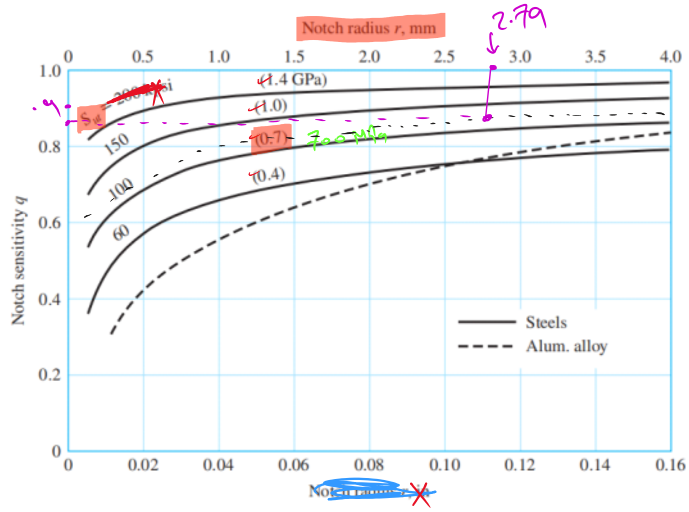


Figure 6–26

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the $r = 0.16$-in (4-mm) ordinate. Source: Sines, George and Waisman, J. L. (eds.), *Metal Fatigue,* McGraw-Hill, New York, 1969.

"q" → notch sensitivity

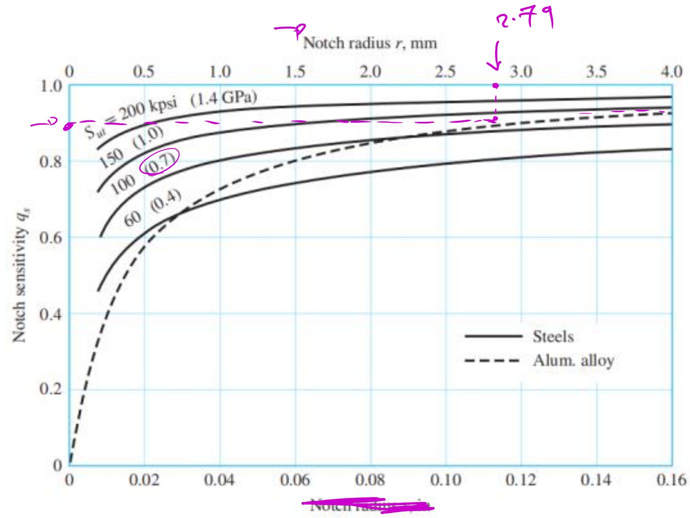
r = 2.79

Sult = 724

q = .85

Figure 6–27

Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of q_s corresponding to $r = 0.16$ in (4 mm).



q_s = .88

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_s(K_{ts} - 1) \tag{6–32}$$

Kf = 1 + .85 (1.68 − 1) = 1.58

Kfs = 1 + .88 (1.42 − 1) = 1.37

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$
$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2} \tag{7–6}$$

Mm = 0
Ma = 142.35 * 10^3
Tm = 124.28 * 10^3
Ta = 0

$A = \sqrt{4(1.58 * 142.35 * 10^3)^2} = 449271.75$

$B = \sqrt{3(1.37 * 124.28 * 10^3)^2} = 294814.36$

Se ⇒ endurance limit

Sult = 724

1 Determine S'_e either from test data or

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \leq 200\ \text{kpsi (1400 MPa)} \\ 100\ \text{kpsi} & S_{ut} > 200\ \text{kpsi} \\ 700\ \text{MPa} & S_{ut} > 1400\ \text{MPa} \end{cases} \quad (6\text{–}10)$$

$S'_e = .5 * S_{ult} = .5 * 724 = 362$ MPa

2 Modify S'_e to determine S_e.

$$S_e = k_a k_b k_c k_d k_e S'_e \quad (6\text{–}17)$$

Ka

$k_a = aS_{ut}^b$

Table 6–2 Curve Fit Parameters for Surface Factor, Equation (6–18)

Surface Finish	Factor a		Exponent b
	S_{ut}, kpsi	S_{ut}, MPa	
Ground	[illegible]	1.38	−0.067
Machined or cold-drawn	[illegible]	3.04	−0.217
Hot-rolled	[illegible]	38.6	−0.650
As-forged	[illegible]	54.9	−0.758

a = 3.04
b = −.217

$K_a = 3.04\,(724)^{-.217} = .729$

Kb

Rotating shaft. For bending or torsion,

$d = 27.94$ mm

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.3 \leq d \leq 2\ \text{in} \\ 0.91d^{-0.157} & 2 < d \leq 10\ \text{in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 7.62 \leq d \leq 51\ \text{mm} \\ 1.51d^{-0.157} & 51 < 254\ \text{mm} \end{cases} \quad (6\text{–}19)$$

For axial,

$$k_b = 1 \quad (6\text{–}20)$$

$K_b = \left(\frac{27.94}{7.62}\right)^{-.107} = .875$

Kc

Nonrotating member. For bending, use Table 6–3 for d_e and substitute into Equation (6–19) for d.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \quad (6\text{–}25)$$

Kc = 1

$$S_T/S_{RT} = 0.98 + 3.5(10^{-4})T_F - 6.3(10^{-7})T_F^2$$
$$S_T/S_{RT} = 0.99 + 5.9(10^{-4})T_C - 2.1(10^{-6})T_C^2 \quad (6\text{–}26)$$

kd

Either use the ultimate strength from Equation (6–26) to estimate S_e at the operating temperature, with $k_d = 1$, or use the known S_e at room temperature with $k_d = S_T/S_{RT}$ from Equation (6–26).

$k_d = 1$

ke

Table 6–4 Reliability Factor k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702

$k_e = .814$

$S_e = k_a k_b k_c k_d k_e S_e'$

$S_e = .729 * .87 * .814 * 362 = 186.75$ MPa

a)

DE-Goodman

$$n = \frac{\pi d^3}{16}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1} \quad (7\text{–}7)$$

$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]^{1/3} \quad (7\text{–}8)$$

DE-Morrow

$$n = \frac{\pi d^3}{16}\left(\frac{A}{S_e} + \frac{B}{\tilde{\sigma}_f}\right)^{-1} \quad (7\text{–}9)$$

$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{\tilde{\sigma}_f}\right)\right]^{1/3} \quad (7\text{–}10)$$

DE-Gerber

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\} \quad (7\text{–}11)$$

$$d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3} \quad (7\text{–}12)$$

DE-SWT

$$n = \frac{\pi d^3}{16}\frac{S_e}{(A^2 + AB)^{1/2}} \quad (7\text{–}13)$$

$$d = \left[\frac{16n}{\pi S_e}(A^2 + AB)^{1/2}\right]^{1/3} \quad (7\text{–}14)$$

A = 44927.75
B = 29814.36
Se = 186.75 MPa
Sult = 724 MPa
d = 27.94

goodman

$n = \frac{\pi * (27.94)^3}{16}\left\{\frac{44927.75}{186.75} + \frac{29814.36}{724}\right\}^{-1}$

n = 1.52

Morrow ($\tilde{\sigma}_f = 1069$)

$n = \frac{\pi * (27.94)^3}{16}\left\{\frac{44927.75}{186.75} + \frac{29814.36}{1069}\right\}^{-1} = 1.6$

$$n = \frac{\pi * (27.94)^3}{16} \left\{ \frac{44927.95}{186.75} + \frac{29814.36}{1069} \right\}^{-1} = 1.6$$

Gerber

$n = 1.73$

swt

$n = 1.38$

f) yield factor of safty "ny"

$$n_y = \frac{S_y}{\sigma'_{max}}$$

$K_f = 1 + .85(1.68 - 1) = 1.58$

$K_{fs} = 1 + .88(1.42 - 1) = 1.37$

$$\sigma'_{max} = [(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2]^{1/2}$$

$$= \left[\left(\frac{32K_f(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2} \qquad (7\text{–}15)$$

Ma ✓ Tm ✓

$$\sigma'_{max} = \left[\left(\frac{32 * 1.58(142.35)}{\pi(27.94)^3}\right)^2 + 3\left(\frac{16 * 1.37 * 124.28}{\pi * (27.94)^3}\right)^2\right]^{1/2}$$

$M_m = 0$
$M_a = 142.35$
$T_m = 124.28$
$T_a = 0$

$\sigma'_{max} = 125.48$ MPa

$S_y = 565$ MPa

$$n_y = \frac{565}{125.48} = 4.5$$

Kt Kts Kf Kfs q qs ⇒ cnG ⇐

$S_e = K_a K_b K_c K_d K_e S_e'$